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COORDINATION SHEET

TO J. C. Turner, 09-07

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GROUP INDEX System Test Operations Support Group

SUBJECT Guidance Systems

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MODEL MM

THEORY OF OPERATION & MECHANIZATION OF THE TYPICAL INERTIAL GUIDANCE SYSTEM

GENERAL

The ICBM missile is a surface to surface, long range, ballistic missile, designed for a low circular probable error.

To achieve this goal, a highly accurate Guidance system must be used. It is the purpose of this document to outline the high points of the operation and mechanization of this type of Guidance system.

THEORY OF OPERATION

The flight of a ballistic missile is characterized by an initial powered portion and a final free-fall of ballistic portion that follows a trajectory passing through the target. For example, between any two points in space, power cutoff and target, there is an infinite number of possible ballistic trajectories each having a different time of flight. To minimize the guidance computation and establish a desirable flight time, a total time of flight from launch to impact is fixed at a predetermined value. The impact area is also determined before launch. The propulsion systems must be capable of providing the Re-entry Vehicle (R/V with a velocity that will be required for a free-fall trajectory to the target.

The velocity having the necessary remaining time to target associated with it, required at any instant to make good a free-fall trajectory passing through the target is termed the correlated velocity (V_c). The guidance system must therefore control thrust direction during flight, and sense missile velocity (V_m) comparing it with (V_c). The difference between V_c and V_m is termed Velocity to be gained (V_g) or $V_c - V_m = V_g$. As the V_g reaches zero the Guidance system must cut off the missiles engines at that precise point and release the R/V on its free fall trajectory to the target.

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To illustrate the above theory, the following figure is presented:

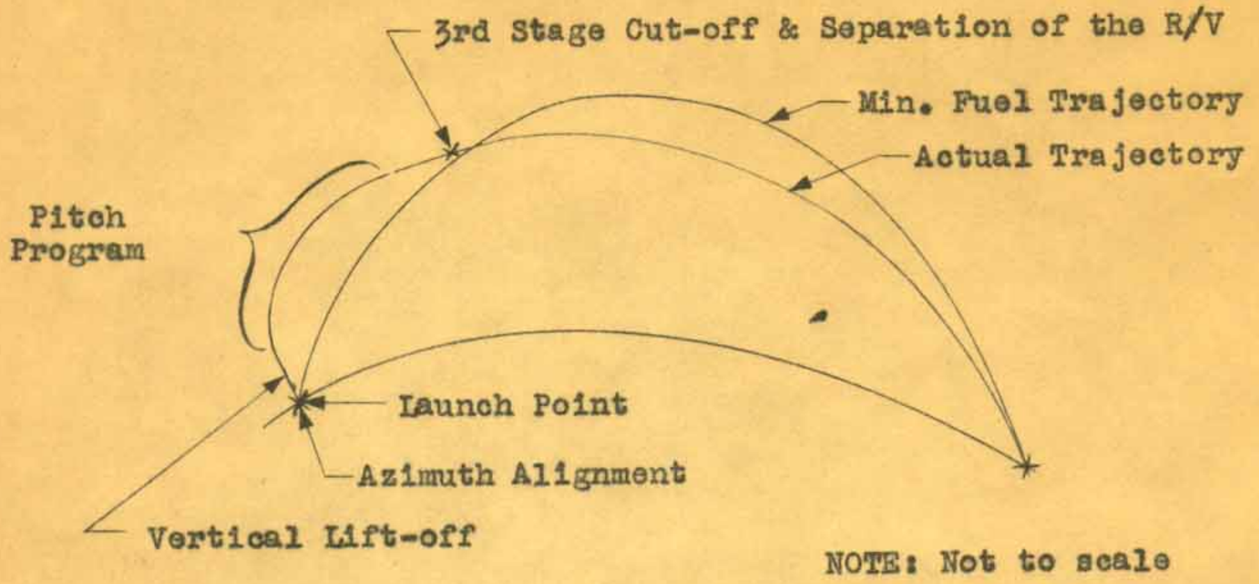


Figure I.

From Figure 1, the minimum fuel trajectory is impractical for two reasons:

a. Because of its weight and aerodynamic instability, the missile must be launched vertically. It would then have to be pitched over immediately to achieve the desired flight angle. This would require the generation of a large angle of attack (acute angle between missile thrust and velocity vectors) which would result in serious bending and side-loading of the airframe and probably reduce the vertical component of thrust to a value smaller than that required to overcome gravity.

b. The missile would remain in the appreciable atmosphere for a longer period of time, and thus generate high velocity at low altitudes. This would cause excessive skin heating.

Both of these effects could be eliminated if the missile were to ascend vertically out of the appreciable atmosphere, but such a flight path would require excessive fuel consumption. A compromise between the two cases is followed in determining the actual trajectory as shown in Figure 1.

Following is an illustration of the ballistic equations involved in such a flight after 3rd stage engine cut-off:

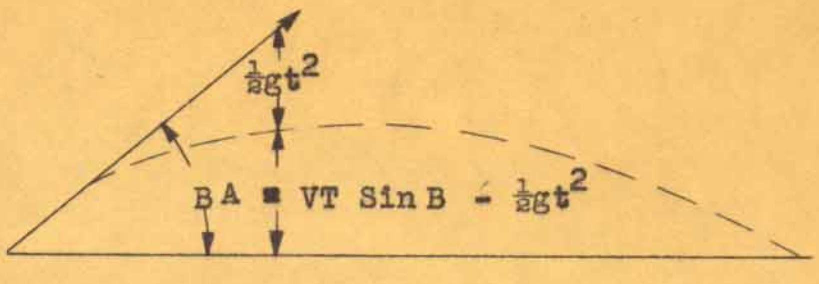


Figure II.

Since a body under the force of gravity (g), falls a distance $1/2\ g^{T^2}$ during any time (T) the altitude (A) of that body may be written as follows:

$A = VT \text{ Sin } B - 1/2\ g^{T^2}$ Equation #1

The range of the body at any time (T) may be expressed as follows:

$R = VT \text{ Cos } B$ Equation #2

The range at the time when the body returns to the datum plane is then the altitude $A = 0$ and the range equation may be written as follows:

$V \text{ Sin } B = 1/2\ g^{T} \quad @\ A = 0$ Equation #3

$T = \frac{2V \text{ Sin } B}{g}$ Equation #4

$R = \frac{V^2 \text{ Sin } 2B}{g}$ by substituting Equation #4 into Equation #2 Equation #5

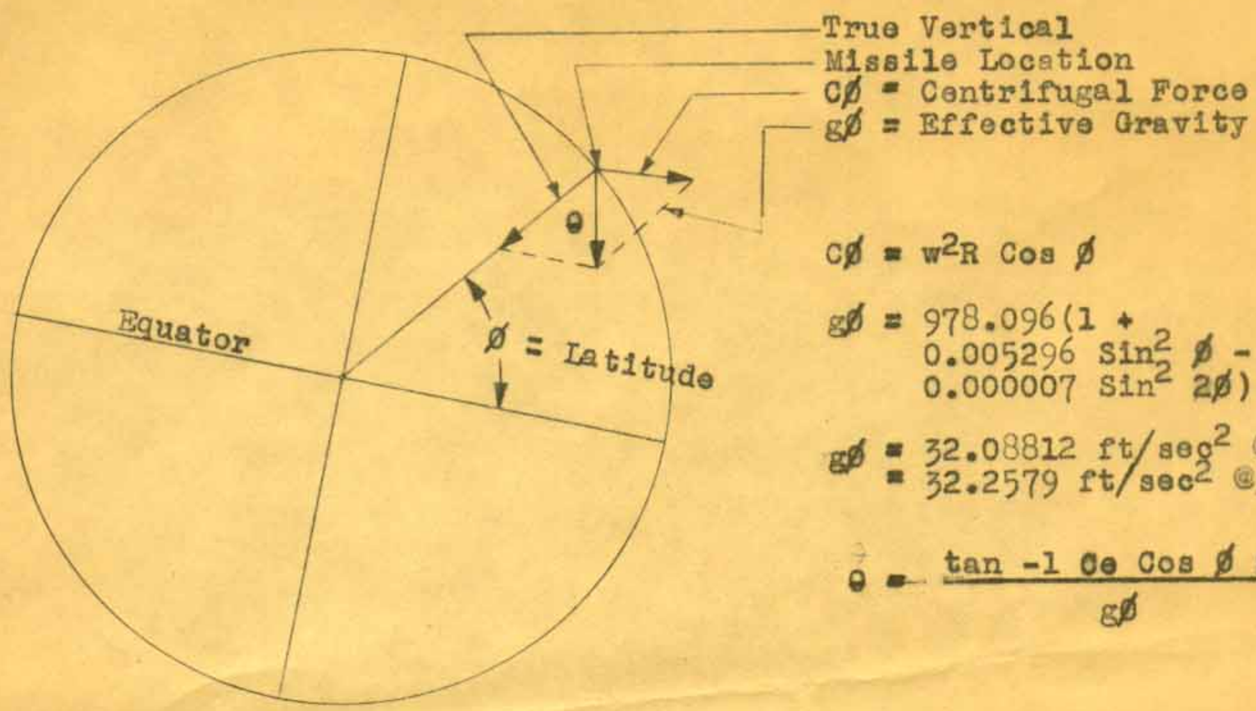
NOTE: The equations listed above neglect the fact that the earth is rotating, hence, range and azimuth adjustments have to be made prior to the free-fall portion of flight.

The effect of earth rotation on a ballistic missile takes place in two forms:

a. Centrifugal Force

Centrifugal force is the force imparted to a missile on the earths' surface by the rotation of the earth. This force has the effects of reducing the effect of gravity (except at north pole) and establishes a new direction of the effective gravitational force.

To clarity this effect the following illustration is presented:



$C\phi = w^2 R \cos \phi$

$g\phi = 978.096(1 + 0.005296 \sin^2 \phi - 0.000007 \sin^2 2\phi)$

$g\phi = 32.08812 \text{ ft/sec}^2 \text{ @} \angle 0^\circ$
$= 32.2579 \text{ ft/sec}^2 \text{ @} \angle 90^\circ$

$$\theta = \frac{\tan^{-1} Ce \cos \phi \sin \phi}{g\phi}$$

Figure III.

Centrifugal force is of interest to the missile engineer, because it represents an error to any pendullos or vertical-indicating device, that must be corrected in order that a true vertical may be indicated. It also introduces an error to the guidance equations gravity factor, that must be corrected in order to obtain the correct correlated velocity to make good a free-fall trajectory to the target.

b. Coriolis Effect

The coriolis effect involves freely falling bodies. This effect is best explained with the aid of the following figure:

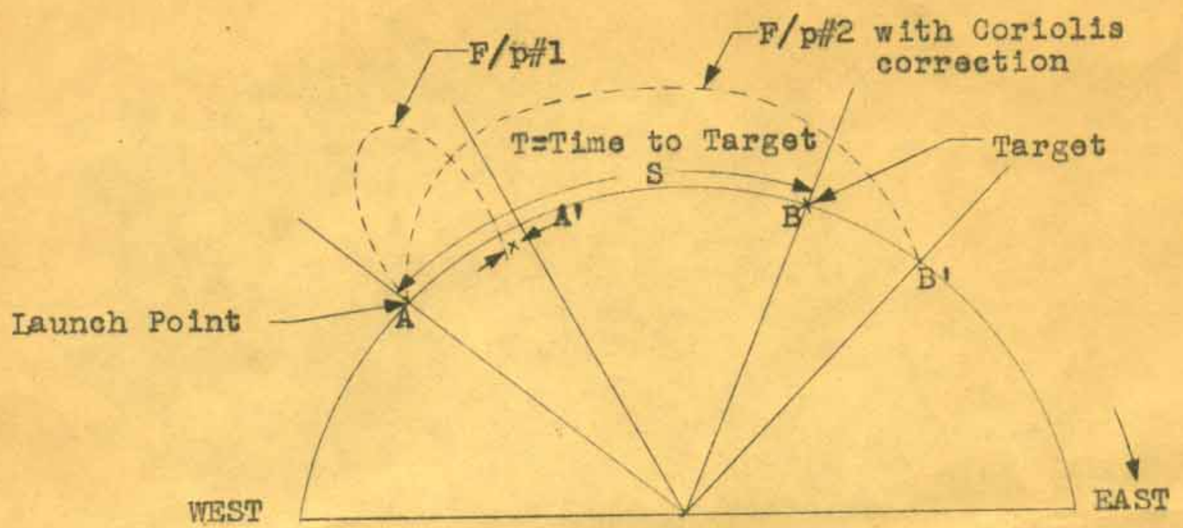


Figure IV.

Flight Path #1

Assume that the missile is launched vertically with a total time of flight equal to the time required to traverse from point A to point B. As the launch Point A moves east, the missile lags behind and falls to earth west of A, which is the new position of the launching point after the time of flight. The deviation between the Point A, and point where the missile falls to earth is termed X.

$$X = \frac{4}{3} wgt^3 \cos \phi$$

where:
w = angular velocity of earth
g = effective gravity
t = time of flight
ϕ = latitude

Flight Path #2

The actual flight of the missile would fall short of the target (Point B) by the value X if the coriolis effect were not taken into account.

Operational Summary

A inertial Guidance System is a lead-reckoning missile guidance system that employs extensive elements which respond to the earth's gravitational field and to inertial effects in accordance with the Newtonian Laws of Motion. The system therefore is not dependent on information obtained from transmitters outside the missile.

TYPICAL INERTIA GUIDANCE SYSTEM MECHANIZATION

A. PRINCIPLES OF STABLE COORDINATE REFERENCE GROUP

The Stable Coordinate Reference Group measures the attitude of the vehicle. This can be considered as actually two types of attitudes:

1. Roll and pitch which indicate the direction of the true vehicle.

2. Heading which measures the direction of true north.

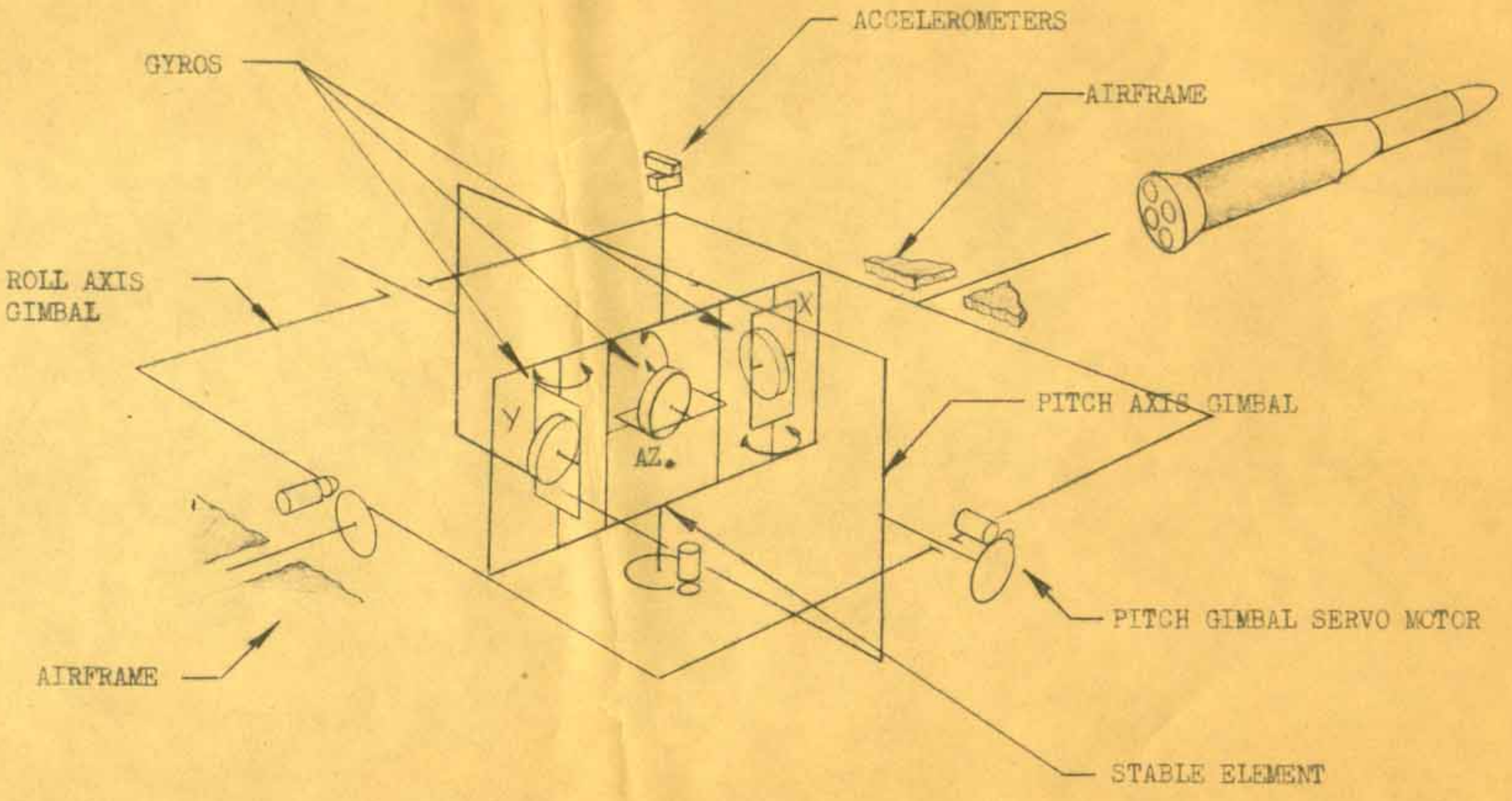
Earlier models of vertical gyros used for roll and pitch information where accurate only for unaccelerated flight. In a slow turn they aligned to a new vertical because the gyros were erected in reference to a pendulum. The earlier heading instruments gave the direction of the earth's magnetic field rather than true north. The disadvantages of using the earth's magnetic field as a reference are that heading data is unreliable in the vicinity of the magnetic poles, and that to correct for magnetic variation at different geographical positions increases equipment complexity.

B. COORDINATE SYSTEM (Figure V)

The well known property of gyroscopes of preserving rigidity in space is utilized in the Stable Coordinate Reference Group. Three "single degrees of freedom" gyros are used. One for pitch, one for roll and one for heading. Two accelerometers are used to indicate X and Y axis acceleration. The example shown below uses three gimbals. Note that one is for roll, one is for pitch, and the third gimbal supports the stable element in which are mounted the three "single degrees of freedom" gyros.

These maintain the platform in its initial orientation. There are three occurrences which if not corrected could make the platform output inaccurate.

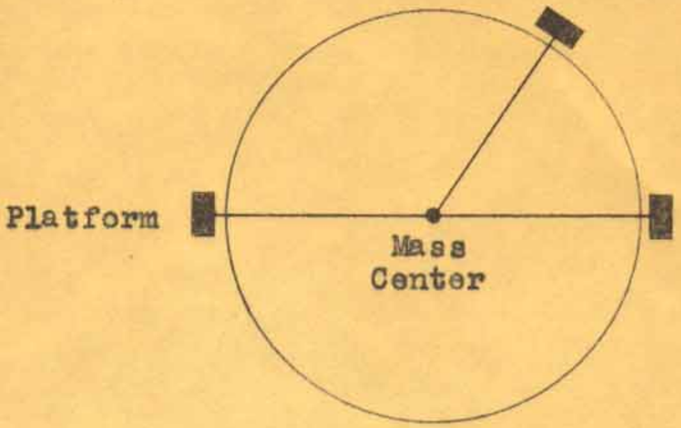
1. The direction of the resultant force on a pendulum or plumb bob changes with accelerations. This can be illustrated by the following example. As soon as a railroad train is set in motion a pendulum suspended in it will swing. The direction of resultant force will not change with accelerations when the length of the pendulum is equal to the radius of the earth. Therefore, this can be simulated by an electrical pendulum that has the same period as a pendulum as long as the radius of the earth, which can be used



GYRO STABLIZED PLATFORM

Figure V.

to keep the platform erect. Therefore, the center of gravity of the platform will be at the center of the earth as shown below:



Center of Mass
Figure VI.

2. The earth turns 360 degrees in 24 hours or 15 degrees each hour. The earth turns at a constant rate of 15 degrees per hour about a polar axis (axis through the north and south poles). Therefore, in order to maintain the platform horizontal with respect to the earth's surface and at a fixed heading, it must be caused to rotate about an axis parallel to the polar axis at 15 degrees per hour.

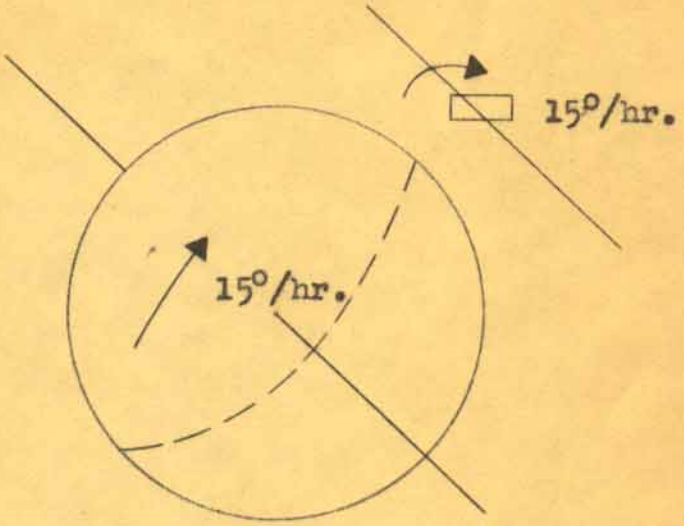


Figure VII.

If the platform is at the equator the rotation must be about a horizontal axis, See Figure VII. If the platform is at one of the poles the rotation must be about a vertical axis, See Figure VIII.

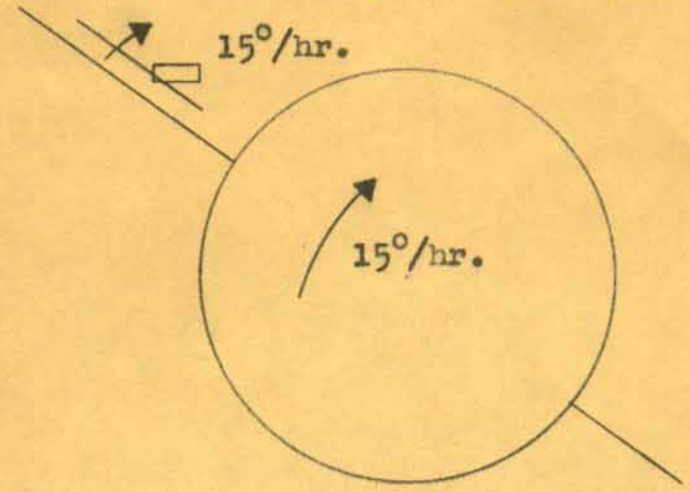


Figure VIII.

Between the equator and poles part of the rotation must be about a horizontal axis and part about a vertical axis. A correction term for latitude is fed into the system to convert the rotation from a horizontal to a vertical axis as the latitude is increased.

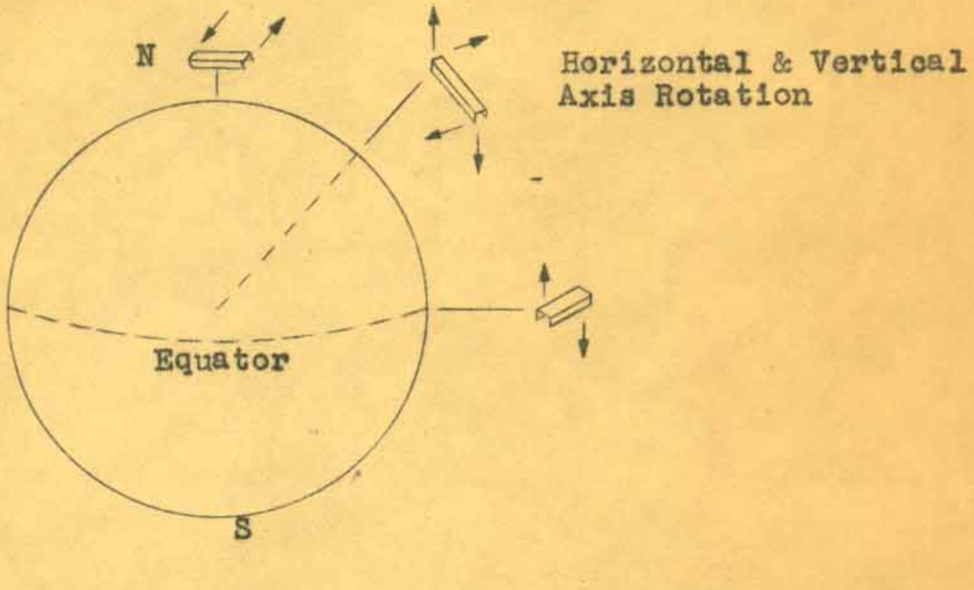
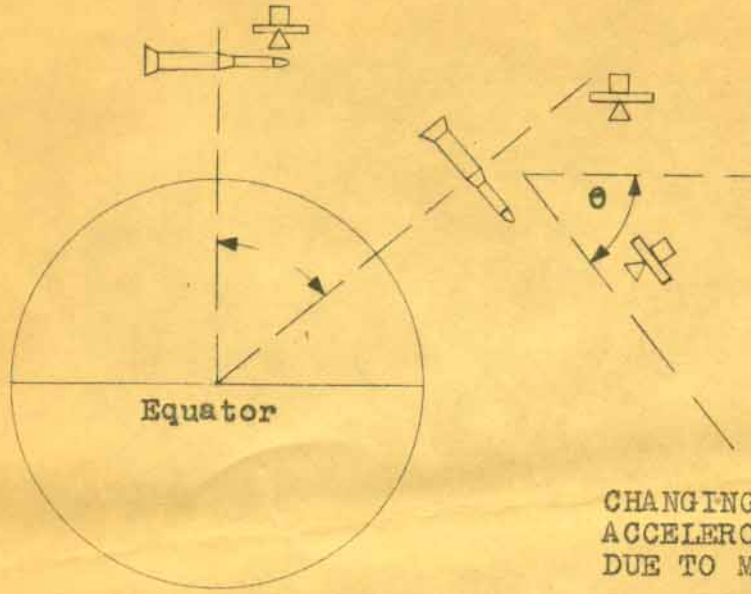


Figure IX.

3. The movement of the vehicle about the surface of the earth must be summed with the compensated earth's turning rate and the platform must be turned to compensate for the fact that the earth's surface is curved. The platform is tipped by applying additional torques to each gyro to show movement over the surface of the earth.

C. STABLE PLATFORM



SPACE STABILIZED ACCELEROMETER PLATFORM IS NO LONGER HORIZONTAL UNLESS----ROTATED THROUGH SAME ANGLE θ AS VEHICLE HAS MOVED ABOUT THE EARTH.

CHANGING OF SPACE STABILIZED ACCELEROMETER PLATFORM FROM HORIZONTAL DUE TO MOVEMENT ABOUT THE EARTH.

Figure X.

1. The purpose of the stable coordinate reference group summerized is:

   a. Maintain pitch and roll gyros erected to the true vertical to the center of the earth.

   b. Provide true north heading and reference and the missile heading.

2. Modern equipment provides a means of measuring each of the above to a high degree of accuracy thus making it possible to utilize a Stable Coordinate Reference Group. Modern missiles are directed by the automatic navigation technique called "Inertial Guidance". The true inertial system is composed of the following:

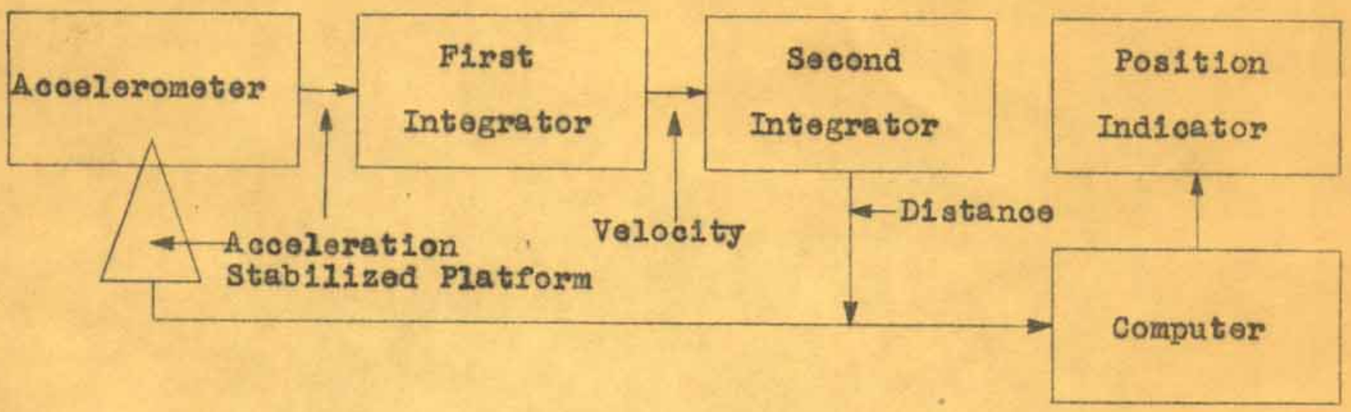
   a. Stabilized Platform

   b. Accelerometer to provide missile accelerations in X and Y axis.

   c. First Integrator to provide velocity.

d. Second Integrator to provide distance.

e. Computer to provide position.



Pure Inertial System
Figure XI.

D. <u>STABLE PLATFORM OPERATION</u> (Figure XII)

In the Initial Mode the Stable Platform is erected to a level position i.e., roll and pitch axis horizontal. The airframe must not be in motion but it need not be level at this time.

4 The integrators are by-passed in the initial mode and, when power is first applied, if the platform is not level, one or both of the accelerometers will put out a signal. Suppose, for example, that the platform is level in roll but not level in pitch, and that the azimuth (inner) gimbal happens to be aligned along the roll axis. See Figure 9. Suppose that the right side of the platform is too high. Then the X accelerometer will have an output to the gravitational attraction of the earth. This signal will be amplified, demodulated and sent to the X gyro torquer causing the gyro gimbal to rotate. The rotation of the gyro will cause an output from the gyro take-off signal generator. This output goes through the resolver on the azimuth gimbal axis and then through an amplifier to the pitch gimbal torquer motor which drives the gimbal toward level (right side down) until the accelerometer ceases to have an output. At this time the platform will be level. For the above example, the gyro take-off signal did not need to be resolved. If, however, the aximuth gimbal had been rotated ninety degrees, i.e., aligned along the pitch axis, and with the initial conditions assumed (right side high), the Y accelerometer would have put out a signal, the Y gyro precessed and the signal would have been resolved through ninety degrees to drive the pitch gimbal torquer motor. Note that the gyroscopic effect did not enter into the operation discussed and this action would have been the same if the X gyro had been replaced by a rigid shaft connecting the motors of the torquer and the signal generator.

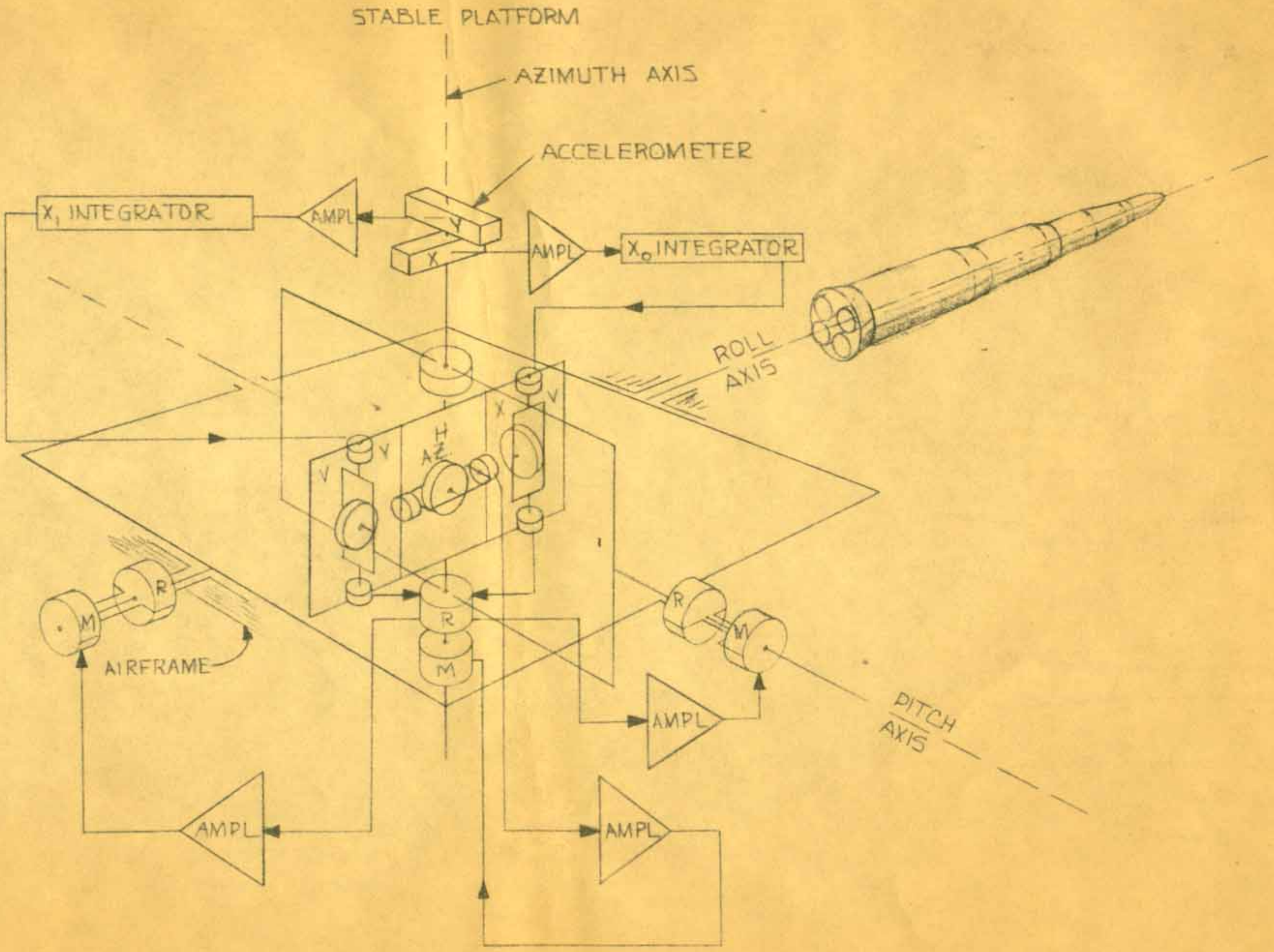

STABLE PLATFORM
AZIMUTH AXIS
ACCELEROMETER
X_1 INTEGRATOR
AMPL
Y
X
AMPL
X_0 INTEGRATOR
ROLL AXIS
V
X
V
Y
V
H
AZ
R
M
R
M
R
M
AIRFRAME
AMPL
AMPL
AMPL
AMPL
PITCH AXIS

Figure XII.

In the Normal Mode the platform must remain level, and maintain its initial heading while the vehicle goes through any flight attitude change.

The three gyros mounted on the platform sense rotation about the three mutually perpendicular axis. If the platform started to rotate clockwise about the azimuth axis, for example, due to turning of the vehicle, the azimuth sensitive gyro would precess, and the gyro take-off signal generator would produce an output which would be amplified and cause the azimuth gimbal torquer motor to drive the azimuth gimbal counter-clockwise. The drive on the azimuth gimbal would be just sufficient to counterbalance the rotation of the vehicle and keep the gyro precession near a null. This would keep the platform heading constant.

The X and Y gyros function in a similar manner except that their outputs are resolved for heading of the platform. The result of the operation in the Normal Mode thus far discussed is to cause the platform to maintain a fixed orientation in space.

In the inertial guidance system, the platform is not maintained at a fixed orientation in space, but rather at a fixed orientation with respect to the earth. It is therefore necessary to compensate for two things:

1. First, superimpose into the other drives, a signal to rotate the platform about an axis parallel to the earth's polar axis at the turning rate of the earth.

2. Then superimpose onto the other drives, a signal to rotate the platform enough so that, as the vehicle moves over the surface of the earth, the platform will follow the curvature of the earth.

The following is a discussion of the second effect. Suppose that the platform is aligned North and South and that the vehicle takes off and travels due North. In this case, the X accelerometer will produce an output, as the vehicle accelerates, which is proportional to the acceleration. This signal is sent to the X integrator. The signal out of the integrator then represents the velocity of the vehicle in the North direction. The turning rate of the vehicle as it stays horizontal and follows the curvature of the earth, is equal to its velocity divided by the radius of the earth. By taking the velocity signal from the integrator and adjusting the scale factor, the velocity signal is made proportional to the vehicles' turning rate. This signal then goes through the X gyro in the same manner that it does in the initial mode to drive the pitch gimbal torquer motor and keep the platform level. The operation is essentially the same for arbitrary headings of the platform and the vehicle except the analysis is more difficult.

MECHANIZATION SUMMARY

Inertial Guidance System mechanization is established in accordance with the guidance equations, which are established by Newtonian Laws of Motion and gravitational effects. The basic subsystems required in this mechanization are a inertial platform and associated electronics, a computer, and a Engine or Nozzle Control Unit.

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